Exam 2 Sp 2006

Part 1, MULTIPLE CHOICE, 5 Points Each

1 A pair of dice are rolled and the sum of the two uppermost faces is recorded. Which of the following sets represents the Sample Space of outcomes for this experiment?

(a) $\{1,2,3,4,5,6\}$ (b) $\{1,2,3,4,5,6,7,8,9,10,11,12\}$ (X) $\{2,3,4,5,6,7,8,9,10,11,12\}$

(d) {all numbers from 1 to 36 inclusive} (e) $\{0,1,2,3,4,5,6\}$

$1+1=2$
$1+2=3$
⋮
$1+6=7$
$2+6=8$
$3+6=9$
⋮
$6+6=12$

0 or 1 are not possible

2 If E and F are events in a sample space, with $Pr(E) = .6$, $Pr(F') = .2$ and $Pr(E \cap F) = .5$, what is $Pr(E \cup F)$?

(a) 1 (b) .2 (c) .8 (d) .1 (X) .9

IN-Ex Principle

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$
$$= .6 + Pr(F) - .5$$
$$= .6 + .8 - .5 = .9$$

Complement Rule.

$$Pr(F) = 1 - Pr(F') = 1 - .2 = .8$$

3 If the odds in favor of the Notre Dame fencing team winning the National Championship this year are 5 to 1, which of the following gives the probability that the Nore Dame Fencing team will win the national Championship this year?

(a) $\frac{1}{5}$ (b) $\frac{5}{6}$ (c) 1 (d) 1.2 (e) $\frac{2}{6}$

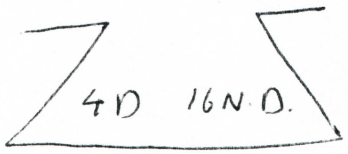
$a:b \rightarrow \frac{a}{a+b}$

odds in favor → Prob Win

$5:1 \rightarrow \frac{5}{6}$

4 A manufacturer of portable compact disc players ships them to retailers in boxes of 20. Before each box is shipped a random sample of 5 C.D. players is taken from the box and tested. If the sample contains no defective players, the box is shipped. If at least one defective player is found in the sample the box is not shipped. What is the probability that a box containing 4 defective C.D. players(and 16 non-defective ones) will pass inspection and be shipped to a retailer?

(a) $\frac{16}{20}$ (b) $1 - \frac{C(16,5)}{C(20,5)}$ (c) $1 - \frac{C(4,1)C(16,4)}{C(20,5)}$ (d) $\frac{C(4,1)C(16,4)}{C(20,5)}$ (e) $\frac{C(16,5)}{C(20,5)}$



Pr(all 5 are N.D.)

$= \frac{\#\text{ Samples will all N.D}}{\text{Total \# Samples of Size 5}}$

$= \frac{C(16,5)}{C(20,5)}$

5 An experiment consists of flipping a coin 10 times. What is the probability of getting at least one head?

(a) $1 - \frac{1}{2^{10}}$ (b) $\frac{1}{2^{10}}$ (c) $\frac{C(10,1)}{2^{10}}$ (d) $\frac{9}{10}$ (e) $1 + \frac{C(10,1)}{2^{10}}$

$Pr(\text{at Least } 1H) = 1 - Pr(\text{No } H)$

$= 1 - \frac{\#\text{ sequences with No H (all T)}}{\text{Total \# Sequences}}$

$= 1 - \frac{1}{2^{10}}$

6 If E and F are events for which $Pr(E) = .5$, $Pr(F) = .4$ and $Pr(E \cup F) = .7$, which of the following gives $Pr(E|F)$.

(a) $\frac{2}{5}$ (b) $\frac{1}{2}$ (c) $\frac{7}{9}$ (d) .2 (e) $\frac{4}{5}$

$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} = \frac{Pr(E \cap F)}{.4} = \frac{.2}{.4} = \frac{1}{2}$

IN - EX

$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$

$.7 = .5 + .4 - Pr(E \cap F)$

$Pr(E \cap F) = .9 - .7 = .2.$

7 A student (Mary Chancy) is taking a multiple choice quiz with 3 questions. Not knowing the material, the student decides to take a random guess at each question. Each question has 5 choices for the answer (labelled (a), (b), (c), (d) and (e)). What is the probability that she will get at least one question correct (assuming she makes independent guesses for each question).

(a) $1-\frac{1}{3^5}$ (b) $1-\frac{1}{5^3}$ (c) $\frac{1}{5}$ (d) $1-\frac{4^3}{5^3}$ (e) $\frac{1}{3^5}$

S = Success = Right answer $\quad F$ = Wrong answer

$Pr(S) = \frac{1}{5}$ $\quad P(F) = 1 - \frac{1}{5} = \frac{4}{5}$.

$Pr(\text{at least one correct}) = 1 - Pr(\text{all wrong})$

$= 1 - \left(\frac{4}{5}\right)^3$

or using binomial Theorem.

$1 - c(3,0)\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^3 = 1-\left(\frac{4}{5}\right)^3$

8 The rules of a carnival game are as follows:

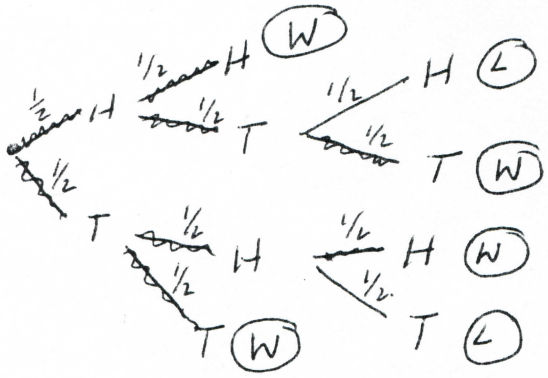
1) You toss a coin at most 3 times (you must stop after 3 tosses)

2) If you get two heads in a row or two tails in a row, you win and you stop playing.

3) If you have tossed the coin 3 times and you have not gotten two heads in a row or two tails in a row, you lose.

What is the probability that you win the game? (A tree diagram might help)

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$ (e) $\frac{3}{8}$



$P(W) = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}$

$= 2\left(\frac{1}{4}\right) + 2\left(\frac{1}{8}\right)$

$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

Questions 9 and 10 both refer to the following data collected from a sample twenty students, recording the number of exams they had in the week prior to spring break:

4, 4, 5, 7, 3, 2, 3, 4, 5, 6, 3, 2, 0, 3, 2, 6, 5, 4, 4, 3.

9 What is the relative frequency of the outcome "2" in the above sample?

(a) $\frac{2}{20}$ (b) 3 (c) $\frac{3}{20}$ (d) $\frac{1}{2}$ (e) $\frac{4}{20}$

n = #observations = 20

#"2"'s = 3

Rel freq. "2" = $\frac{3}{20}$

Obs.	freq
0	1
1	0
2	3
3	5
4	5
5	3
6	2
7	1
	20

10 What is the sample mean for the above sample?

(a) 3 (b) 3.35 (c) 3.5 (d) 4 (e) 3.75

$\bar{x} = 0 \cdot \frac{1}{20} + 1 \cdot \frac{0}{20} + 2 \cdot \frac{3}{20} + 3 \cdot \frac{5}{20} + 4 \cdot \frac{5}{20} + 5 \cdot \frac{3}{20} + 6 \cdot \frac{2}{20} + 7 \frac{1}{20}$

$\frac{75}{20} = 3.75$

Part II, PARTIAL CREDIT,

Show all of your work for credit

11, (10 pts) A pair of dice, red and green are rolled and the pair of uppermost faces is observed.

(a) How many outcomes are there in the sample space (all possible pairs) for this experiment?

$\underset{R}{6} \times \underset{G}{6} = 36$

$\{(1,1)_{RG}\ (1,2)_{RG} \cdots (1,6)_{RG}$

$\vdots$

$(6,1)_{RG}\ (6,2)_{RG} \cdots (6,6)_{RG}\}$

(b) Let E be the event "The sum of both numbers is 4". Which subset of the sample space corresponds to the event E?

$E = \{(1,3)_{RG}, (2,2)_{RG}\ (3,1)_{RG}\}$

(c) What is $Pr(E)$? Equally Likely Sample Space

$Pr(E) = \frac{\#E}{\text{Total \# in SS}} = \frac{3}{36} = \frac{1}{12}$

(d) If F is the event "At least one of the dice shows a 6", are E and F mutually exclusive. Give a reason for your answer.

Same as

Is $E \cap F = \emptyset$

$E = \{(1,3)\ (2,2)\ (3,1)\}$.

F = at least one 6. $= \{(6,1)(6,2) \cdots (6,6) \cdots\}$

everything has a 6

so $E \cap F = \emptyset$

and Yes E and F are Mutually Exclusive.

12, (10 pts) Recall that a Poker hand is a hand of 5 cards dealt randomly from a deck of 52. The Deck has 4 suits(Hearts, Diamonds, Clubs and Spades) and 13 Denominations (A's, K's, Q's, ..., 2's).

(a) If a poker hand is dealt randomly from such a deck, what is the probability that is consists entirely of spades?

13 spades

$$Pr(\text{all spades}) = \frac{\#\text{ hands Spades}}{\text{Total \# hands}} = \frac{C(13,5)}{C(52,5)}$$

(b) What is the probability of getting a Royal Flush (an Ace, King, Queen, Jack and 10 from the same suit)?

$$Pr(\text{Royal Flush}) = \frac{\#\text{ Royal Flushes}}{\text{Total \# Poker hands}}$$

only 4 possible Royal Flushes
Hearts
spades
clubs
Diamonds.

$$= \frac{4}{C(52,5)} = \frac{4}{2,598,960}$$

(c) What is the probability of getting a Royal Flush Consisting of Spades?

$$Pr(\text{Royal Flush of Spades}) = \frac{\#\text{ Royal Flushes of spades}}{\text{Total \# Poker Hands}} = \frac{1}{C(52,5)} = \frac{1}{2,598,960}$$

(d) Use the above results to determine the probability of Getting a Royal Flush or a hand consisting entirely of spades.

RF

S

RF or S

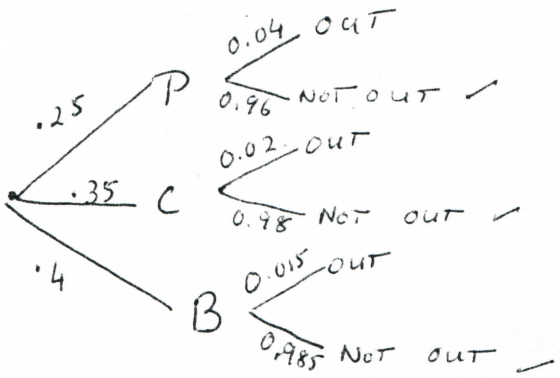
$$Pr(RF \cup S) = Pr(RF) + Pr(S) - Pr(RF \cap S)$$

$$= \frac{4}{C(52,5)} + \frac{C(13,5)}{C(52,5)} - \frac{1}{C(52,5)}$$

$$= \frac{4 + 1287 - 1}{2,598,960} = \frac{1290}{2,598,960}$$

$$\approx .000496.$$

13 (12 pts) (a) A college science library consists of 25% physics books, 35% chemistry books and 40% biology books. The chances of a physics, chemistry or biology book being checked out at any given time are 0.04, 0.02 and 0.015 respectively. If the librarian picks a book title at random from the library catalogue, what is the probability that it is NOT checked out(A tree diagram might help)?



$P(\text{NOT OUT})$
$= (.25)(.96) + (.35)(.98) + (.4)(.98)$
$= .24 + .343 + .394$
$= .977$

(b) In a class of 80 people, all but 10 will leave campus for spring break. Of the 70 who will leave, each student has either purchased a plane ticket or will go to Florida or both. Fifty of the students who are leaving have purchased a plane ticket and forty will go to Florida. What is the probability that a student is going to Florida given that they have purchased a plane ticket?

P = Those who have purchased a plane Ticket
F = " " will go to Florida.

$n(u) = 80$
$n(P) = 50$
$n(F) = 40$
$n(P \cup F) = 70$

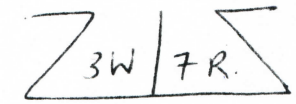
$n(P \cup F) = n(P) + n(F) - n(P \cap F)$
$70 = 50 + 40 - n(P \cap F)$

$n(P \cap F) = 90 - 70 = 20$

$$P_r(F|P) = \frac{P_r(F \cap P)}{P_r(P)} = \frac{n(F \cap P)}{n(P)} = \frac{20}{50} = \frac{2}{5}$$

14, (12 pts) An Urn contains 3 white balls and 7 red balls. A sample of two balls is chosen at random from the urn and the number of white balls in the sample is recorded (It could be 0, 1, or 2).

(a) Find the probabilities for each of the above outcomes and write out the probability distribution for this experiment.



Sample Size 2

$P(\#W = 0) = P(2R)$

$= \frac{\#\text{ Samples with } 2R}{\text{Total } \#\text{ samples}}$

$= \frac{C(7,2)}{C(10,2)} = \frac{21}{45}$

$= .467$

#W	0	1	2
Prob	$\frac{21}{45}$	$\frac{21}{45}$	$\frac{3}{45}$

$Pr(1W) = Pr(1W1R) = \frac{C(3,1)C(7,1)}{C(10,2)}$

$= \frac{21}{45}$

$P(2W) = \frac{C(3,2)}{C(10,2)} = \frac{3}{45}$

(b) If the rules of a carnival game are as follows:

1) You pay \$1 to play

2) You pull out a sample of two balls from the Urn (described at the beginning of the question).

3) You get paid \$1 for every white ball in the sample.

Write out the probability distribution for the earnings for the player of this game.

Earnings

$0W \rightarrow \$0 - \$1 = \$-1$

$1W \rightarrow \$1 - \$1 = 0$

$2W \rightarrow \$2 - \$1 = \$1$

#W	0	1	2
\$ Earnings	-1	0	1
Prob.	$\frac{21}{45}$	$\frac{21}{45}$	$\frac{3}{45}$

(c) What are the expected earnings for the player for this game.

$\text{Expected Earnings} = \sum \text{earnings} \times \text{Prob}$

$= -1 \cdot \frac{21}{45} + 0 \cdot \frac{21}{45} + 1 \cdot \frac{3}{45}$

$= \frac{-21+3}{45} = \frac{-18}{45} = -\$.4 = -40¢$

15, (6 pts) (a) An electronic device has 4 transistors that operate independently. For each transistor the probability of failure in the first 5,000 hours of use is .1. What is the probability that at least one of the transistors will still be working after 5,000 hours of use?

$$\Pr(\text{At Least one working})$$
$$= 1 - \Pr(\text{all Fail})$$
$$= 1 - \left[\Pr(1^{st}\text{ Fails}) \cdot \Pr(2^{nd}\text{ Fail}) \cdot \Pr(3^{rd}\text{ Fails}) \cdot \Pr(4^{th}\text{ Fails})\right]$$
$$= 1 - [(.1)(.1)(.1)(.1)]$$
$$= 1 - .0001 = .9999$$

16 For two extra points sign the following:

I Netty The Incredible will drive safely over break and I will wear my sunscreen in the sun.